4.8 Solving Absolute Value Equations

What you should learn:

Goal 1 How to solve and check absolute value equations using algebra

Goal 2 How to use a graph to check solutions of absolute value equations

Why you should learn it:

You can use absolute value equations to model many real-life situations, such as representing minimum and maximum amounts.

Goal 1 Solving and Checking Algebraically

You already know how to use mental math to solve some absolute value equations. For instance, you know that the equation $|x| = 8$ has *two* solutions: 8 and -8.

Expression inside absolute value sign can be positive.	**Expression inside absolute value sign can be negative.**
8 is a solution of $|x| = 8$ because $|8| = 8$.	-8 is a solution of $|x| = 8$ because $|-8| = 8$.

To solve a more general absolute value equation, use the fact that the expression inside the absolute value sign can be either positive or negative.

Example 1 *Solving an Absolute Value Equation*

Solve $|x - 2| = 5$.

Solution

$$
\begin{aligned}
|x-2| &= 5 && \textit{Rewrite original equation.}\\
x-2 = 5 \text{ or } x-2 &= -5 && \textit{Expression can be 5 or } -5.\\
x = 7 \text{ or } \quad x &= -3 && \textit{Add 2 to both sides.}
\end{aligned}
$$

The equation has two solutions: 7 and -3. Check these solutions by substituting each into the original equation. ■

Example 2 *Isolating the Absolute Value Expression*

Solve $|2x - 7| - 5 = 4$.

Solution First, isolate the absolute-value portion on one side of the equation.

$$
\begin{aligned}
|2x-7| - 5 &= 4 && \textit{Rewrite original equation.}\\
|2x-7| &= 9 && \textit{Add 5 to both sides.}\\
2x-7 = 9 \text{ or } 2x-7 &= -9 && \textit{Expression can be 9 or } -9.\\
2x = 16 \text{ or } \quad 2x &= -2 && \textit{Add 7 to both sides.}\\
x = 8 \text{ or } \quad x &= -1 && \textit{Divide both sides by 2.}
\end{aligned}
$$

The equation has two solutions: 8 and -1. Check these solutions by substituting each into the original equation. ■

Goal 2 Using a Graphic Check

In Lesson 4.6, you learned to use a graphic check for solutions of linear equations. You can use the same technique to check solutions of absolute value equations.

- Rewrite the one-variable equation so that the right side is zero.
- Set y equal to the left side and sketch the graph of the resulting two-variable equation.
- The x-intercepts of the graph of the two-variable equation should match the solutions of the one-variable equation.

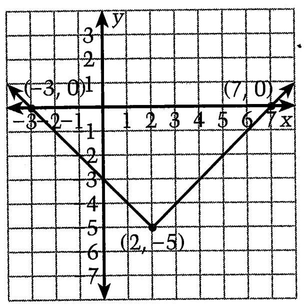
Example 3 *Using a Graphic Check*

Check the solutions of $|x - 2| = 5$ graphically.

Solution In Example 1, the solutions were found to be 7 and -3. You can check these solutions graphically as follows.

$|x - 2| = 5$ ***Original one-variable equation***

$|x - 2| - 5 = 0$ ***Rewrite with zero on right side.***



Set y equal to the left side of the rewritten equation.

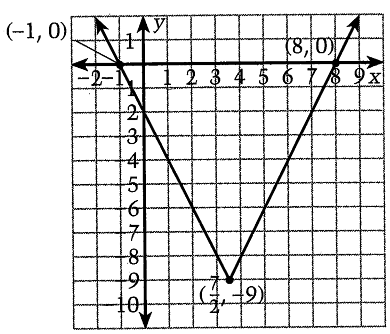
$y = |x - 2| - 5$ ***Form two-variable equation.***

Using the techniques in Lesson 4.7, you know that the graph of this equation is V-shaped and has a vertex at $(2, -5)$. From the graph, you can see that the x-intercepts occur at 7 and -3, which match the solutions. ■

Example 4 *Using a Graphic Check*

Check the solutions of $|2x - 7| - 5 = 4$ graphically.

Solution In Example 2, the solutions were found to be 8 and -1. You can check these solutions graphically as follows.



$|2x - 7| - 5 = 4$ ***Original one-variable equation***

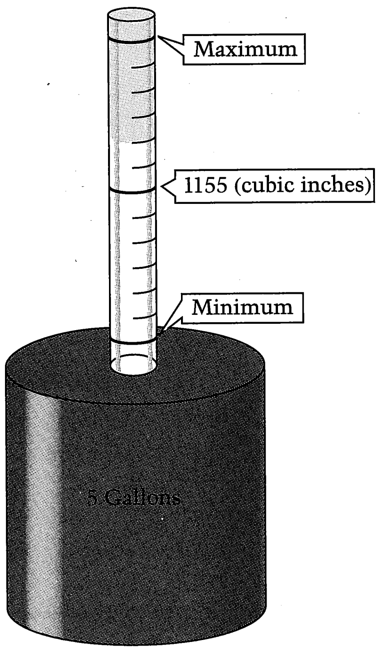
$|2x - 7| - 9 = 0$ ***Rewrite with zero on right side.***

Set y equal to the left side of the rewritten equation.

$y = |2x - 7| - 9$ ***Form two-variable equation.***

The graph of this equation is V-shaped, opens up, and has a vertex at $(\frac{7}{2}, -9)$. From the graph, you can see that the x-intercepts occur at 8 and -1, which match the solutions. ■

Real Life
Weights and Measures



Example 5 *Accuracy of Measurement*

To obtain certification by the Bureau of Weights and Measures, a gasoline pump must pump within 6 cubic inches of 5 gallons (1155 cubic inches) when its gauge reads 5 gallons. The inspector pumps gasoline into a container like that shown at the left. The reading on the gas pump is 5 gallons. To be certified, the gas level in the container must fall within the minimum and maximum amounts on the scale. What are these minimum and maximum amounts? Find an absolute value equation that models this situation and has the minimum and maximum amounts as its solutions.

Solution Using the information given with the photo, the amount pumped must be within 6 cubic inches of 1155 cubic inches. Thus, the minimum and maximum amounts are:

Minimum

$$1155 - 6 = 1149 \text{ cubic inches}$$

Maximum

$$1155 + 6 = 1161 \text{ cubic inches}$$

These two numbers can be expressed as solutions of the equation $|x - 1155| = 6$. Do you see why? ■

Communicating about ALGEBRA

SHARING IDEAS about the Lesson

Interpret a Model Solve each equation. Use the solutions to rephrase the statement.

A. $|x - 160| = 1$ To pass certification, the scale must weigh within one ounce of 160 ounces.

B. $|x - 189| = 86$ To be eligible for one of the 13 weight classes on the wrestling team, your weight must be within 86 pounds of 189 pounds.

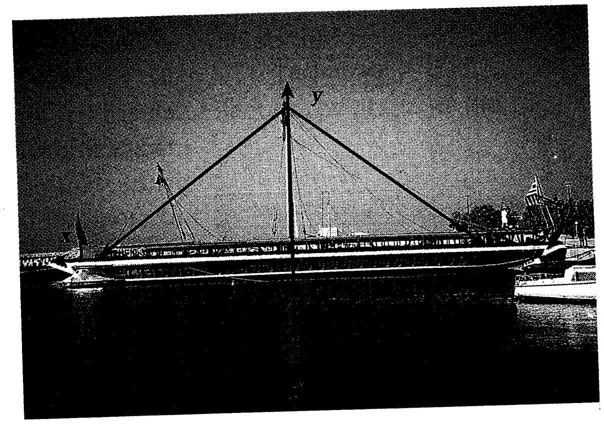
C. $|x - 30| = 10$ To obtain a bonus, the pizza delivery driver must deliver the pizza in 30 minutes, plus or minus 10 minutes.

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

1. Solve $|x + 1| = 2$.
2. What equation could you graph to check the solutions of $|x + 1| = 2$?
3. Which two equations would you solve to find the solutions of $|x + 1| = 32$?
4. If a coordinate plane is drawn through the galley at the right, the mainstays would lie on the graph of $y = 82.5 - 0.92|x|$, where x and y are measured in feet. How far above the deck does the mast rise? Find the x-coordinates of the points at which the mainstays are attached to the deck.



Around 650 B.C., the Greeks invented a warship called the trireme. It had three rows of oars. The photo shows a modern replica of a trireme.

Independent Practice

In Exercises 5–10, solve the equation algebraically.

5. $|x + 4| = 3$
6. $|9 - x| = 4$
7. $|6 - x| = 9$
8. $|x + 12| = 8$
9. $|2x + 6| = 14$
10. $|3x - 4| = 7$

In Exercises 11–13, rewrite the equation so that the term involving absolute value is isolated on one side.

11. $6 + |x + 1| = 9$
12. $|x - 17| + 9 = 1$
13. $7 - |4 - x| = 12$
14. Use a graphic check that -2 is a solution of $|x - 5| = 7$.
15. Use a graphic check that -13 is a solution of $|x + 4| = 9$.

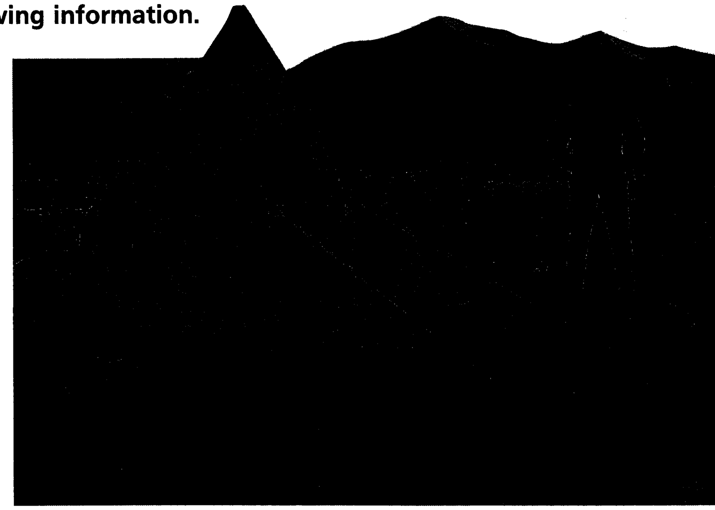
In Exercises 16–24, solve the equation.

16. $|x + 1| = 7$
17. $|x - 1| = 7$
18. $|3x - 2| - 2 = 5$
19. $|4x + 3| - 4 = 8$
20. $3 + |-2x + 9| = 10$
21. $6 + |-x - 5| = 9$
22. $|4 - 5x| - 8 = 15$
23. $|7 - 3x| - 10 = 4$
24. $2|3x - 7| + 2 = 4$

In Exercises 25–33, solve the equation. Use a graphic check.

25. $|x - 9| = 4$
26. $|x + 7| = 16$
27. $|2x - 4| + 6 = 9$
28. $4 + |x - 10| = 14$
29. $10 + |3x + 1| = 24$
30. $|2x + 9| - 15 = 36$
31. $2|x + \frac{1}{2}| - 1 = 9$
32. $3|x + \frac{1}{6}| + 5 = 17$
33. $3|4x + 9| - 2 = 6$

A Tent on Ice **In Exercises 34–37, use the following information.**



If a coordinate plane were drawn through the tent at the right, the edges of the tent would lie on the graph of $y = -2|x - 5| + 10$, where x and y are measured in feet. The ropes that anchor the tent lie on the graph of $y = -\frac{1}{3}|x - 5| + 6$.

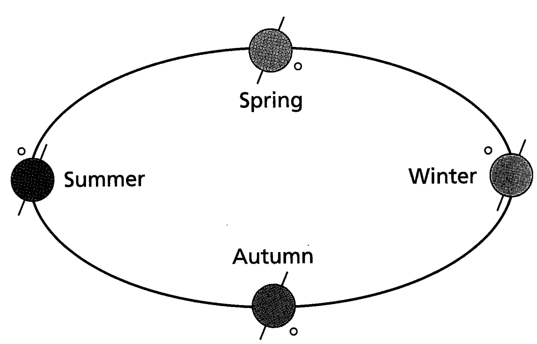
34. How tall is the tent?

35. How wide is the tent at its base?

36. Find the x-coordinates of the points at which the stakes are secured to the ice.

37. How far are the stakes from the edge of the tent?

38. ***Earth's Orbit*** Because Earth travels in an elliptical orbit around the sun, the distance between Earth and the sun varies. The average distance is 92,950,000 miles, but the distance can vary 1,550,000 miles from the average. Find an absolute value equation that models this situation and has the minimum and maximum distances between Earth and the sun as solutions. What are the minimum and maximum distances?



39. ***Boxing Weights*** A junior heavyweight boxer's weight must fall within 7 pounds of 183 pounds. Find an absolute value equation that models this situation and has the minimum and maximum weights for a junior heavyweight boxer as solutions. What are the minimum and maximum weights?

Integrated Review

In Exercises 40–45, evaluate the expression.

40. $2|4| + 6$

41. $3 - |8 - 4|$

42. $3^2 - |-4|$

43. $6^2 - 2|14 - 19|$

44. $|3 - 9| \cdot 2^2 + 1$

45. $-4|-5 + 1| - 12$

46. Write an equation of the horizontal line passing through $(-2, 4)$.

47. Write an equation of the vertical line passing through $(5, -3)$.

48. Find the slope and y-intercept of $x + 2y = 18$.

49. Find the slope and y-intercept of $6x - y = 3$.

50. Sketch the line whose slope is $\frac{3}{2}$ and whose y-intercept is -5.

51. Sketch the line whose slope is -4 and whose y-intercept is 12.